DO NOW

Worksheet 4.7.1 Answers:

1. 55 and 55	6. 8 ft and 8 ft
2. -4 and 4	7. 300 m by 600 m
3. 24 and 8	8. 4 in by 4 in by 1 in
4. 50 and 25	9. 5/3 inches
5. 25 ft and 25 ft	10. 6 in by 6 in by 3 in

4.7 Optimization Problems - Day 2

Procedure:

1. Identify all ***given*** quantities and quantities ***to be determined***. If possible, make a sketch.
2. Write a **primary equation** for the quantity that is to be maximized or minimized.
3. Reduce the primary equation to one having a ***single independent variable***. This may involve the use of **secondary equations** relating the independent variables of the primary equation.
4. Determine the feasible domain of the primary equation.
5. Find the derivative of the primary equation and find its critical number
6. Use the first and/or second derivative tests to determine the maximum minimum. CLEARLY identify the appropriate answer(s).

Example:

4. On a given day, the flow rate F (in cars per hour) on a congested roadway is $F = \frac{v}{22 + 0.02v^2}$. Where v is the speed of the traffic in miles per hour. What speed will maximize the flow rate on the road?

$F = \frac{v}{22+0.02v^2}$ domain: $x>0$

$F' = \frac{(22+.02v^2)(1) - v(.04v)}{(22+0.02v^2)^2}$

$F' = \frac{22+.02v^2 - .04v^2}{(22+.02v^2)^2}$

$F' = \frac{22-.02v^2}{(22+.02v^2)^2}$

$22-.02v^2=0$ $\cancel{22+.02v^2=0}$ D.N.E.

$v^2 = \frac{22}{.02}$

$v^2 = 1100$

$v = \pm\sqrt{1100} = 10\sqrt{11}$

rej. neg.

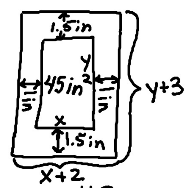
$(0, 10\sqrt{11})$ | $(10\sqrt{11}, \infty)$

$f'(1) = +$ | $f'(50) = -$

maximum

$\boxed{10\sqrt{11} \text{ mph or approximately } 33 \text{ mph}}$

5. A page is to contain 45 square inches of print. The margins at the top and bottom of the page are each 1.5 inches wide. The margins on each side are 1 inch. What should be the dimensions of print so that a minimum amount of paper is used?



domain: $x>0$

$A=(x+2)(y+3)$

$A=(x+2)(\frac{45}{x}+3)$

$A=45+3x+\frac{90}{x}+6$

$A=51+3x+90x^{-1}$

$A'=3-\frac{90}{x^2}$

$A'=\frac{3x^2-90}{x^2}$

$3x^2-90=0$ $\cancel{x^2=0}$ not in domain

$x^2=30$

$x=\pm\sqrt{30}$

rej. negative

$xy=45$

$y=\frac{45}{x}$

$y=\frac{45}{\sqrt{30}}$

$y=\frac{45\sqrt{30}}{30} = \frac{3\sqrt{30}}{2}$

$(0,\sqrt{30})$ | $(\sqrt{30},\infty)$

$f'(1) = -$ | $f'(6)=+$

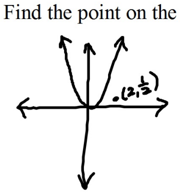
minimum

$x=\sqrt{30}$

$\boxed{\sqrt{30} \text{ in by } \frac{3\sqrt{30}}{2}}$

6. Find the point on the graph of $y = x^2$ that is closest to the point (2, ½).

↑Secondary



$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$D=\sqrt{(x-2)^2+(y-\frac{1}{2})^2}$

$D=\sqrt{(x-2)^2+(x^2-\frac{1}{2})^2}$

$D=\sqrt{x^2-4x+4+x^4-x^2+\frac{1}{4}}$

$D=\sqrt{x^4-4x+4.25}$

minimize the radicand

$R=x^4-4x+4.25$

$R'=4x^3-4$

$4x^3-4=0$

$4(x^3-1)=0$

$x=1$

$R''=12x^2$

$R''(1)=+$ ↑ minimum

$x=1$

$\therefore y=1$ $(y=x^2)$

$\boxed{(1,1)}$

HOMEWORK

Worksheet - HW 4.7.2